UNIT – VI

Time Varying Fields :-

The Equations describing relationships between time varying Electric & magnetic fields are known as Maxwells Equations. Time varying fields or dynamic fields are produced due to time varying currents.

⇒ Faraday's law & lenz's law :-

According to faradays law, a static magnetic field can not produce any current flow. But with time varying fields, an electromotive force induces which may drive a current in a closed path or circuit.

Statement of Faradays law :-

"The electro motive force induced in a closed path or circuit is proportional to rate of change of magnetic flux enclosed by closed path".

He observed that when a closed path moves in a magnetic field, current is generated and hence emf. The same observations he made with closed path kept fixed and magnetic field was varied. The effect is commonly called electro magnetic Induction.

$$\therefore \quad e = -N\frac{d\phi}{dt} \longrightarrow (1)$$

N = No. of turns in circuit
e = induced emf

Assume single turn circuit ie N = 1 then

$$e = -\frac{d\phi}{dt}$$

statement of Lenz's law :- The -ve sign in the induced emf according to faraday was explained by Lenz's law

" The direction of induced emf is such that it opposes the cause producing it ie. changes in magnetic flux".

$$\therefore \quad e = -N\frac{d\phi}{dt}$$

where -ve sign indicates induces emf opposes the cause.

let us consider faradays law, the induced emf is a scalar quantity measured in volts & it is given by

$$e = \oint \bar{E}\cdot\bar{dL} \longrightarrow (2)$$

The induced emf in equation (2) indicates a voltage about a closed path ie any part of the closed path changes emf also changes".

let the magnetic flux passing through a specified area is given by $\phi = \int_s \bar{B}\cdot\bar{ds}$ $\quad \bar{B}$ = flux density

Substitute in equation of faradays law

$$\Rightarrow e = -\frac{d\phi}{dt} = -\frac{d}{dt}\int_s \bar{B}\cdot\bar{ds} \longrightarrow (3)$$

From (2) and (3)

$$\therefore \quad e = \oint \bar{E}\cdot\bar{dL} = -\frac{d}{dt}\int_s \bar{B}\cdot\bar{ds}$$

(2)

The variation of flux ϕ with time t can be caused due to any of the following conditions.

i, By having stationary closed path in time varying field $\bar{B}$

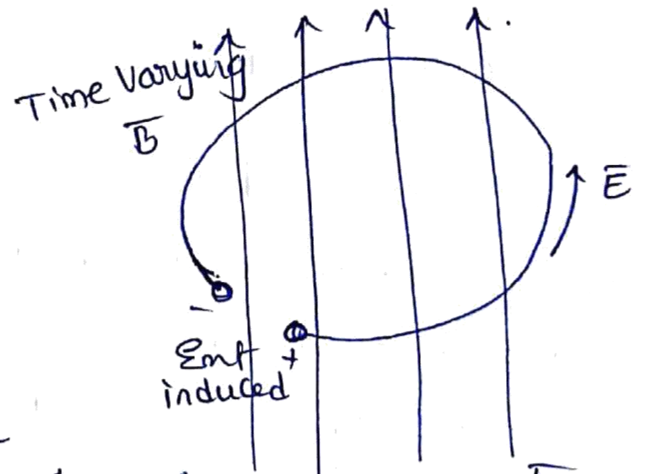
ii, By having a time varying closed path in a static field $\bar{B}$

iii, By having time varying closed path in time varying field $\bar{B}$

In the first case induced emf is called statically induced emf, in second case emf induced is called dynamically induced emf

⇒ <u>Statically Induced Emf</u> / <u>Transformer Emf</u>

A statically closed path in a time varying $\bar{B}$ field.



According Faradays law we have

$$e = \oint \bar{E}\cdot\bar{dl} = -\frac{d}{dt}\int_s \bar{B}\cdot\bar{ds}$$

In this case conductor is not time varying and the flux is varying with respect to time hence we can write

$$\oint \bar{E}\cdot\bar{dl} = -\int_s \frac{\partial \bar{B}}{\partial t}\cdot\bar{ds}$$

This is similar to transformer action & emf is called transformer emf.

Apply stokes theorm

$$\oint_L \bar{E}\cdot \bar{dl} = \int_S (\nabla\times\bar{E})\cdot \bar{ds}$$

$$\therefore \quad \int_S (\nabla\times\bar{E})\cdot\bar{ds} = -\int_S \frac{\partial \bar{B}}{\partial t}\cdot\bar{ds}$$

Assume both surface integrals taken over identical surfaces then

$$\boxed{\nabla\times\bar{E} = -\frac{\partial\bar{B}}{\partial t}}$$

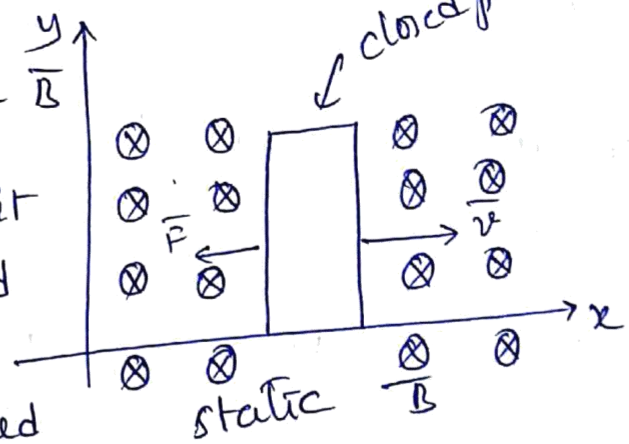
This Equation is Called Maxwells 4th Equation.

If $\bar{B}$ is not varying then $\frac{\partial\bar{B}}{\partial t} = 0$

$$\boxed{\nabla\times\bar{E} = 0}$$

⇒ Dynamically induced emf / motional emf / Generator emf :-

A moving closed path in static $\bar{B}$



when a closed path or circuit is moving in a static $\bar{B}$ field an emf is induced in the closed path. This emf is called dynamically induced emf

Consider that a charge Q is moved in a magnetic field $\bar{B}$ at a velocity $\bar{v}$. Then the force on charge is given by $F = Q\,\bar{v}\times\bar{B}$

But motional electric field is given by $\bar{E}_m = \frac{\bar{F}}{Q} = \bar{v}\times\bar{B}$

Thus the induced Emf is given by.

$$\oint \bar{E}_m \cdot \bar{dl} = \oint (\bar{v} \times \bar{B}) \cdot \bar{dl}$$

This is Emf induced when a closed path moving in field $\bar{B}$.

⇒ when moving closed path is placed in a Time varying $\bar{B}$:-

when a moving closed path placed in a time varying $\bar{B}$, Then both statically & dynamically induced Emf. are present.

∴ The induced Emf in This Case is sum of transformer Emf & motional Emf

$$\oint \bar{E} \cdot \bar{dl} = \int \frac{\partial \bar{B}}{\partial t} \cdot \bar{ds} + \oint (\bar{v} \times \bar{B}) \cdot \bar{dl}$$

⇒ Displacement Current density

For static electromagnetic fields, according to Ampere's law, we can write

$$\nabla \times \bar{H} = \bar{J} \longrightarrow (1)$$

Taking divergence on both sides

$$\nabla \cdot (\nabla \times \bar{H}) = \nabla \cdot \bar{J}$$

But according to vector identity "divergence of The curl of any vector field is zero".

$$\nabla \cdot (\nabla \times \bar{H}) = \nabla \cdot \bar{J} = 0 \longrightarrow (2)$$

But from the Equation of continuity

$$\nabla \cdot \bar{J} = -\frac{\partial \rho_v}{\partial t} \longrightarrow (3)$$

∴ From Equation (2) it is clear that

$\frac{\partial \rho_v}{\partial t} = 0$, They only Equation (2) becomes true.

Thus Equations (2) & (3) are not compatible for time varying fields. we must modify Equation (1) by adding one unknown term Say $\bar{N}$

∴ Equation becomes $\nabla \times \bar{H} = \bar{J} + \bar{N}$

Again apply divergence on both Sides

$$\nabla \cdot (\nabla \times \bar{H}) = \nabla \cdot \bar{J} + \nabla \cdot \bar{N} = 0$$

As $\nabla \cdot \bar{J} = -\frac{\partial \rho_v}{\partial t}$ ∴ $\nabla \cdot \bar{N} = -\nabla \cdot \bar{J}$

$$\boxed{\nabla \cdot \bar{N} = \frac{\partial \rho_v}{\partial t}}$$

According to Gauss law

$$\rho_v = \nabla \cdot \bar{D}$$

$$\Rightarrow \nabla \cdot \bar{N} = \frac{\partial}{\partial t}(\nabla \cdot \bar{D}) = \nabla \cdot \frac{\partial \bar{D}}{\partial t}$$

Compare two sides

$$\bar{N} = \frac{\partial \bar{D}}{\partial t}$$

∴ Now we can write Ampere's circuital law

$$\nabla \times \bar{H} = \bar{J}_c + \frac{\partial \bar{D}}{\partial t}$$

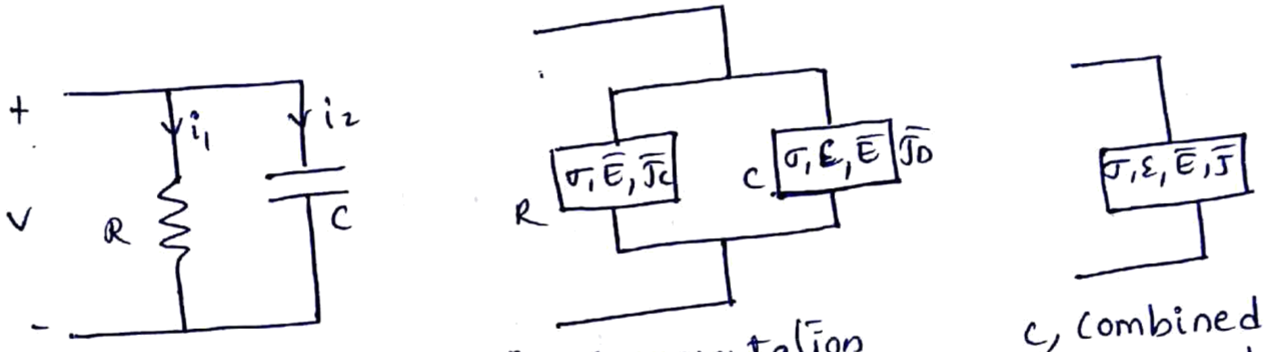
J_c = conduction current density

$\frac{\partial \bar{D}}{\partial t} = J_D$ = displacement Current density

$$\Rightarrow \boxed{\nabla \times \bar{H} = \bar{J}_c + \bar{J}_D}$$

⇒ physical Significance of displacement Current

Consider that the parallel RC combination is driven by the time varying ie sinusoidal voltage V.



physical parallel RC circuit

Box representation of two elements

c, combined element

Current through resistor is $i_1 = \frac{V}{R}$

This current is called conduction current, because it is due to the flow of electrons & is indicated by i_c.

Let A be the cross-sectional area of resistor.

$$\therefore \quad \bar{J}_c = \frac{i_c}{A} = \sigma \bar{E}$$

Now Assume initial charge on a capacitor is zero. Then for time varying ~~fields~~. voltage applied across parallel plate capacitor. Then

$$i_2 = C \frac{dv}{dt}$$

Let two plates of area A are seperated by distance d with dielectric having permittivity ε in between plates

$$i_2 = \frac{\varepsilon A}{d} \frac{dv}{dt} \longrightarrow (1)$$

This current is called displacement current denoted by i_D. & E produced by voltage applied is

$$E = \frac{V}{d} \Rightarrow V = Ed.$$

Substitute V in (1)

$$\Rightarrow i_D = i_2 = \frac{\varepsilon A}{d} \cdot \frac{d}{dt}(dE)$$

$$= \frac{\varepsilon A}{\cancel{d}} \cdot \cancel{d} \frac{dE}{dt} \quad \& \quad E = \frac{D}{\varepsilon}$$

$$\Rightarrow i_D = \varepsilon A \cdot \frac{d}{dt}\left(\frac{D}{\varepsilon}\right)$$

$$= \cancel{\varepsilon} A \cdot \frac{1}{\cancel{\varepsilon}} \cdot \frac{dD}{dt}$$

$$i_D = A \cdot \frac{dD}{dt}$$

Now displacement current density

$$\bar{J}_D = \frac{i_D}{A}.$$

$$\bar{J}_D = \frac{\cancel{A} \frac{d\bar{D}}{dt}}{\cancel{A}}$$

$$\Rightarrow \boxed{\bar{J}_D = \frac{\partial \bar{D}}{\partial t}}$$

$$\therefore \text{Total } \boxed{\bar{J} = \bar{J}_c + \bar{J}_D}$$

we have $\bar{J} = \bar{J}_c + \bar{J}_D$

$$= \sigma \bar{E} + \frac{\partial \bar{D}}{\partial t}$$

for electric field $\bar{E}$, let the time dependence be given by $e^{j\omega t}$. Then

$$\bar{J} = \sigma \bar{E} + \frac{\partial}{\partial t}(\varepsilon \bar{E})$$

$$\bar{J} = \sigma \bar{E} + j\omega\varepsilon \bar{E}$$

Then the ratio of conduction current density magnitude to the magnitude of displacement current density is given by.

$$\boxed{\frac{|\bar{J}_c|}{|\bar{J}_D|} = \frac{\sigma}{\omega\varepsilon}}$$

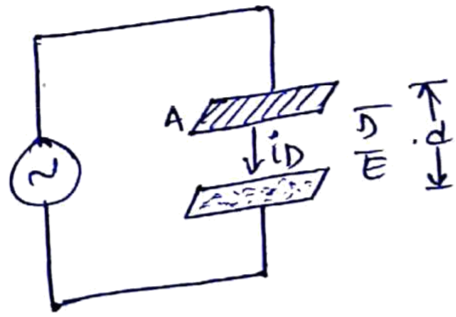
if $\frac{\sigma}{\omega\varepsilon} >> 1$ medium is conductor

$\frac{\sigma}{\omega\varepsilon} << 1$ medium is dielectric

⇒ prove that the displacement current in dielectric of parallel plate capacitor is equal to the conduction current in the leads

Consider parallel plate capacitor is connected to time varying voltage V.

$$V = V_m \sin \omega t$$



The current through the leads the capacitor is conduction current & is given by

$$i_c = C\frac{dv}{dt} = C\frac{d}{dt}(V_m \sin \omega t)$$

$$i_c = (C\omega)V_m \sin \omega t \longrightarrow (1)$$

The displacement current is the current flowing through the dielectric between parallel plates.

$$i_D = \int_S \overline{J_D} \cdot \overline{ds}$$

$$i_D = \int_S \frac{\partial \overline{D}}{\partial t} \cdot \overline{ds}$$

$\overline{D}$ & $\overline{ds}$ are ~~pe~~ perpendicular to plates and hence in same direction.

$$\therefore i_D = \int_S \frac{\partial D}{\partial t} ds.$$

$$= \int_S \frac{\partial \varepsilon E}{\partial t} ds$$

$$= \varepsilon \int_S \frac{\partial E}{\partial t} ds$$

$$= \varepsilon \int_S \frac{\partial}{\partial t}\left(\frac{V}{d}\right) ds \qquad \because E = \frac{V}{d}$$

$$= \frac{\varepsilon}{d} \cdot \int_S \frac{\partial V}{\partial t} ds$$

$$= \frac{\varepsilon}{d} \frac{\partial V}{\partial t} \cdot \int_S ds.$$

$$= \frac{\varepsilon}{d} \cdot \frac{\partial V}{\partial t} \cdot A$$

$$i_D = \frac{\varepsilon A}{d} \cdot \frac{\partial V}{\partial t}$$

$$i_D = C \cdot \frac{\partial V}{\partial t} \qquad \because C = \frac{\varepsilon A}{d}$$

$$\Rightarrow i_D = C \cdot \frac{\partial}{\partial t}(V_m \sin\omega t) = C \cdot V_m \omega \cos\omega t$$

$$i_D = (C\omega) V_m \cos\omega t \quad \rightarrow (2)$$

From (1) & (2) it is clear that current through dielectric of capacitor is equals to the current through the leads.

Maxwells Equations for static fields

Maxwells Equations are derived from Faradays law, Ampere circuit law, Gauss's law for electro static fields & Gauss's law for magnetostatic fields.

a. Maxwells Equation derived from Faradays law :-

For electro static fields, The work done over a closed path is always zero.

$$\therefore \oint \bar{E} \cdot \bar{dl} = 0 \longrightarrow (1)$$

Equation (1) is called integral form of Maxwell's Equation derived from Faradays law for static field.

using stokes theorm converting the closed line integral in to Surface integral.

$$\oint \bar{E} \cdot \bar{dl} = \int_s (\nabla \times \bar{E}) \cdot \bar{ds} = 0$$

$$\therefore \int_s (\nabla \times \bar{E}) \cdot \bar{ds} = 0 \qquad \bar{ds} \neq 0 \text{ hence}$$

$$\boxed{\nabla \times \bar{E} = 0} \longrightarrow (2)$$

This Equation is called point or differential form of Maxwell's Equation derived from Faradays law for static fields.

b, Maxwell's Equation derived from Ampere's circuit law

For magnetostatics an Ampere's circuital law states that line integral of field intensity H around a closed path is exactly Equal to current enclosed by that path.

ie $\oint \overline{H}\cdot\overline{dl} = I \longrightarrow (3)$

above Equation is called

$\Rightarrow$ we know $I = \int_s \overline{J}\cdot\overline{ds}$

$$\Rightarrow \oint \overline{H}\cdot\overline{dl} = \int_s \overline{J}\cdot\overline{ds}$$

above Equation is called integral form of maxwell's Equation derived from Ampere's circuit law for static field.

apply stokes theorm to LHS

$$\Rightarrow \oint \overline{H}\cdot\overline{dl} = \int_s (\nabla\times\overline{H})\cdot\overline{ds} = \int_s \overline{J}\cdot\overline{ds}$$

hence $\nabla\times\overline{H} = \overline{J}$

This Equation is called point or differential form of maxwell's Equation derived from Ampere's law for static field.

c) Maxwell's Equation derived from Gauss's law for Electrostatic fields :-

According to Gauss law for electrostatic field, the electric flux passing through any closed surface is Equal to the total charge enclosed by that surface.

$$\psi = \oint \overline{D}\cdot\overline{ds} = Q_{enclosed}$$

The most common form to represent Gauss law, is with volume charge density ρ_v

$$\oint \overline{D}\cdot\overline{ds} = \int_v \rho_v \cdot dv \longrightarrow (4)$$

Equation (4) is called integral form of maxwell's Equation derived from Gauss's law for static electric field.

Apply divergence theorm on ~~RHS~~. LHS.

$$\oint_s \overline{D} \cdot \overline{ds} = \int_v (\nabla \cdot \overline{D}) dv = \int_v \rho_v dv$$

$$\Rightarrow \int_v (\nabla \cdot \overline{D}) dv = \int_v \rho_v dv$$

If integration is applied over identical volume elements then $\boxed{\nabla \cdot \overline{D} = \rho_v}$

This Equation is called point or differential form of maxwell's Equation derived from electrostatic fields.

D) Maxwell's Equation derived from Gauss law for magnetostatic fields :-

According to Gauss law for magnetostatic field we have. magnetic flux cannot reside in a closed surface due to non existance of single magnetic pole.

$$\oint_s \overline{B} \cdot \overline{ds} = 0$$

This is called integral form of maxwell's Equation derived from Gauss law for static magnetic field.

Apply divergence theorm

$$\oint_s \overline{B} \cdot \overline{ds} = \int_v (\nabla \cdot \overline{B}) dv = 0$$

$$\Rightarrow \boxed{\nabla \cdot \overline{B} = 0}$$ This is point form of Maxwell Equation derived from Gauss law.

****⟹* Maxwell's Equations for time varying Fields

Maxwells Equations are derived from Faradays law Ampere's circuit law., Gauss's law for electric field and Gauss law for magnetic field.

A, Maxwell's Equation derived from Faraday's law

From faradays law which relates Emf induced in a circuit to the time rate of decrease of total magnetic flux linking the circuit

$$\oint \bar{E}\cdot d\bar{l} = -\int_s \frac{\partial \bar{B}}{\partial t}\cdot d\bar{s}$$

This is the maxwell's Equation derived from Faraday's law. Expressed in integral form.

Statement :- "The total electromotive force (emf) induced in a closed path is Equal to negative Surface integral of the rate of change of flux density with respect to time over the entire Surface bounded by same closed path".

using stokes Theorm., convert line integral to surface integral ⇒ $\oint_L \bar{E}\cdot d\bar{l} = \int_s (\nabla\times\bar{E})\cdot d\bar{s} = -\int_s \frac{\partial \bar{B}}{\partial t}\cdot ds$

Assuming that integration is taken over identical Surface then $\boxed{\nabla\times\bar{E} = -\frac{\partial \bar{B}}{\partial t}}$

This is maxwell's Equation derived from Faradays law Expressed in point form or differential form.

B, Maxwells Equation derived from Ampere's circuit law:

According to Ampere's law, The line integral of magnetic field intensity H around a closed path is Equal to current enclosed by the path.

$$\oint \bar{H} \cdot \bar{dl} = I_{enc}$$

Replacing current interms of current density

$$I_{enc} = \int_s \bar{J} \cdot \bar{ds}$$

$$\Rightarrow \quad \oint \bar{H} \cdot \bar{dl} = \int_s \bar{J} \cdot \bar{ds}$$

Above Expression can be made further general by adding displacement current density

$$\oint \bar{H} \cdot \bar{dl} = \int_s \left[\bar{J} + \frac{\partial D}{\partial t} \right] \cdot \bar{ds}$$

This is maxwell's Equation derived from Ampere's circuit law. & it is called integral form of maxwell Equation

Statement: "The total magnetomotive force around any closed path is Equal to the surface integral of the conduction and displacement current densities over the entire surface bounded by the same closed path.

Applying Stokes Theorm. on LHS

$$\oint \bar{H} \cdot \bar{dl} = \int_s (\nabla \times \bar{H}) \cdot \bar{ds} = \int_s \left[\bar{J} + \frac{\partial \bar{D}}{\partial t} \right] \cdot \bar{ds}$$

If the integration is applied over identical surface then we can write

$$\nabla \times \bar{H} = \bar{J}_c + \frac{\partial \bar{D}}{\partial t}$$

above Equation is called point form of maxwell's Equation derived from Ampere's circuit law.

c, Maxwell's Equation derived from Gauss law for electric fields

According to Gauss law, the total flux out of the closed surface is equal to the net charge with in the surface

$$\oint_s \bar{D} \cdot \bar{ds} = Q_{enc}$$

we can write Q_{enc} interms of volume charge density as $Q_{enc} = \int_v \rho_v \cdot dv$

$$\Rightarrow \boxed{\oint_s \bar{D} \cdot \bar{ds} = \int_v \rho_v \, dv}$$

This Equation is called maxwell's Equation for electric fields derived from Gauss's law Expressed in integral form ~~and~~

Statement : "The Total flux leaving out of a closed surface is equal to total charge enclosed by a finite volume."

using divergence theorm. $\int_v (\nabla \cdot \bar{D}) dv = \int_v \rho_v dv$

$$\Rightarrow \oint_s \bar{D} \cdot \bar{ds} = \int_v (\nabla \cdot \bar{D}) dv$$

$$\Rightarrow \int_v (\nabla \cdot \bar{D}) \, dv = \int_v \rho_v \, dv$$

⑨

If the integration is applied over identical volume elements

then we can write $\boxed{\nabla \cdot \bar{D} = \rho_v}$

above Equation is called point form of maxwell's Equation derived from Gauss law.

D, maxwell's Equation derived from Gauss's law magnetic fields

for magnetic fields, the surface integral of $\bar{B}$ over a closed surface s is always zero due to non existence of monopole.

$$\oint_S \bar{B} \cdot d\bar{s} = 0$$

This is maxwell's Equation expressed in integral form.

statement :- "The surface integral of magnetic flux density over closed surface is always Equals to zero.

using divergence theorem convert surface integral to Volume integral

$$\oint_S \bar{B} \cdot d\bar{s} = \int_V (\nabla \cdot \bar{B})\, dv = 0.$$

$$\Rightarrow \int_V \nabla \cdot \bar{B}\, dv = 0 \qquad \text{as } dv \neq 0$$

$$\Rightarrow \boxed{\nabla \cdot \bar{B} = 0}$$

This is point form of maxwell's Equation derived from Gauss law for magnetic fields.

Maxwell's Equations & Significance

~~For magnetic fields, the surface integral of B over a closed surface is is always zero, due to no~~

Differential form	Integral form.	Significance
$\nabla \times \bar{E} = \frac{-\partial \bar{B}}{\partial t}$	$\oint \bar{E} \cdot \bar{dl} = -\int_S \frac{\partial \bar{B}}{\partial t} \cdot \bar{ds}$	Faraday's law
$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$	$\oint \bar{H} \cdot \bar{dl} = \int_S \left(\bar{J} + \frac{\partial \bar{D}}{\partial t}\right) \cdot \bar{ds}$	Ampere's circuital law
$\nabla \cdot \bar{D} = \rho_v$	$\oint_S \bar{D} \cdot \bar{ds} = \int_S \rho_v \, dv$	Gauss's Law
$\nabla \cdot \bar{B} = 0$	$\oint_S \bar{B} \cdot \bar{ds} = 0$	No isolated magnetic charges

poynting Theorm and poynting vector :-

Time varying fields or dynamic fields Constitute the electro magnetic waves (Like electro magnetic waves in radio waves), they travel through the free Space or a dielectric. In electro magnetic waves, the power and energy relationships Can be explained in terms of amplitudes of electric and magnetic fields.

The resulting theorm is the most fundamental relationships of the electromagnetic theory which is known as poynting theorm.

By means of electro magnetic waves, an energy can be transported from transmitter to receiver. The energy stored in an electric field & magnetic field is transmitted at a certain rate of energy flow, which can be calculated with the help of poyting theorm.

Electric field expressed in V/m

magnetic field expressed in A/m.

As the product of these two fields gives $\left(\frac{V}{m}\times\frac{A}{m}=\frac{watt}{m^2}\right)$ a new vector called as power density & it the product of $\bar{E}$ and $\bar{H}$ and power density itself is a ~~po~~ vector quantity as it has particular direction

$$\therefore \quad \bar{P}=\bar{E}\times\bar{H} \qquad watt/m^2$$

where $\bar{P}$ is called poynting vector, & it is the instantaneous power density vector associated with electro-magnetic field (EM) at a given point.

The poynting theorm is based on law of conservation of energy in electromagnetism.

poyting theorm can be stated as

" The net power flowing out of a given volume 'V' is equals to the time rate of decrease in energy stored within volume 'V' minus the ohmic power dissipated.

Suppose $\bar{E}=E_x\bar{a}_x$, $\bar{H}=H_y\bar{a}_y$

$$\therefore \bar{P}=\bar{E}\times\bar{H}= E_x\bar{a}_x\times H_y\bar{a}_y = P_z\bar{a}_z$$

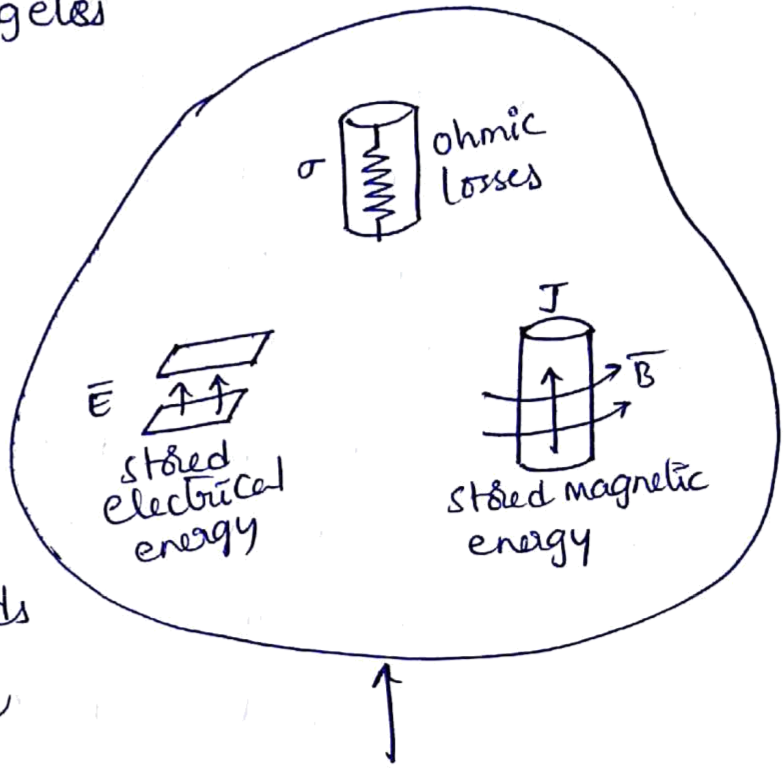
$\therefore$ $\bar{E}, \bar{H}$ & $\bar{P}$ are perpendicular to each other

Consider electric field propagates in free Space.

$$\bar{E} = [E_m \cos(\omega t - \beta z)]\, \bar{a}_x$$

Magnetic field

& $\bar{H} = (H_m \cos(\omega t - \beta z))\, \bar{a}_y$



In the medium, The ratio of magnitudes of E & H depends on intrinsic impedance η

$$\eta = \eta_0 = \frac{E_m}{H_m} = 120\pi = 377\Omega$$

moreover in free Space, electromagnetic wave travels at a Speed of light

$$\therefore \quad \bar{H} = [H_m \cos(\omega t - \beta z)]\, \bar{a}_y$$

$$\bar{H} = \left[\frac{E_m}{\eta_0} \cdot \cos(\omega t - \beta z)\right] \bar{a}_y$$

According to poynting theorm.

$$\bar{P} = \bar{E} \times \bar{H} = E_m \cos(\omega t - \beta z)\, \bar{a}_x \times \left[\frac{E_m}{\eta_0} \cos(\omega t - \beta z)\right] \bar{a}_y$$

$$\boxed{\bar{P} = \frac{E_m^2}{\eta_0} \cos^2(\omega t - \beta z)\, \bar{a}_z \quad W/m^2}$$

This is power density measured in watt/m². Thus power passing through area is given by

power = power density × Area

Average power density :

$$P_{avg} = \frac{1}{T}\int_0^T \frac{E_m^2}{\eta} \cos(\omega t-\beta z)\, dt$$

$$= \frac{E_m^2}{T\eta}\int_0^T \frac{1+\cos 2(\omega t-\beta z)}{2}\, dt$$

$$= \frac{E_m^2}{T\eta}\left[\frac{t}{2} + \frac{\sin 2(\omega t-\beta z)}{2(2\omega)}\right]_0^T$$

$$= \frac{E_m^2}{T\eta}\left[\frac{t}{2} + \frac{\sin(2\omega t-2\beta z)}{4\omega}\right]_0^T$$

$$= \frac{E_m^2}{T\eta}\left[\frac{T}{2} + \frac{\sin(2\omega T-2\beta z)}{4\omega} - \frac{\sin(-2\beta z)}{4\omega}\right]$$

But $\omega T = 2\pi$

$$\therefore P_{avg} = \frac{E_m^2}{T\eta}\left[\frac{T}{2} + \frac{\sin(4\pi-2\beta z)}{4\omega} + \frac{\sin 2\beta z}{4\omega}\right]$$

$$= \frac{E_m^2}{T\eta}\left[\frac{T}{2} - \cancel{\frac{\sin 2\beta z}{4\omega}} + \cancel{\frac{\sin 2\beta z}{4\omega}}\right]$$

$$= \frac{E_m^2}{2\eta}$$

Hence average power is given by

$$P_{avg} = \frac{1}{2}\frac{E_m^2}{\eta} \quad W/m^2$$